Sets

Well defined collection of elements or numbers

Types of Sets

① Null Set :- The set which contain no elements at all is called empty, null or void set
Denoted by 'ϕ' or '{ }'
ex :- $\{x : x \neq x\}$

② Singleton set :- The set which contain only one element is called singleton set ex :- $\{x : x+5 = 8\}$ or $x = \{3\}$

③ finite set :- The set which contain finite or limited no. of element is finite set. In other words we can count the different element of a set
ex :- $\{x : 1 < x < 7$ and $x \in N\}$
$\{2, 3, 4, 5, 6\}$

④ Infinite set :- The set which is not finite is infinite set ex :- N, I, R

⑤ Subset :- Let A and B be two sets

if every element of A is also an element of B then A is called a subset of B and it is denoted by $A \subseteq B$ and readed as A as subset of B

$A \subseteq B \Rightarrow \{x \in B \Rightarrow x \in B\}$

If A is not a subset of B :- $A \notin B$

ex :- $A\{2,4,6\}$; $B\{2,4,6,8\}$

$\Rightarrow A \subseteq B$

⑥ Proper subset :- Let A and B be two sets then A is called a proper subset if

- A is a subset of B i.e $A \subseteq B$ and
- there exist at least one element at B which doesn't belongs to A and it is denoted by '$A \subset B$' when A is a proper subset of B then B is called superset of A and denoted by '$B \supset A$'

⑦ Improper subset :- Set A is called improper subset of B if and only if $A = B$
Every set is a improper subset of itself or empty set is proper subset of every set $A \neq \phi$

⑧ Equal set :- Every element of A is also an element of B $\Rightarrow A \subseteq B$
Every element of B is also an element

of A $B \subseteq A$.

(9) **Power set :-** let A be a set of all subset of A is called the power set of A it is denoted by $\mathcal{P}(A)$

ex:- (1) $A = \{2,3\}$

$\mathcal{P}(A) = \{\phi, \{2\}, \{3\}, \{2,3\}\}$

(2) $A = \{2,3,4,5\}$

$\mathcal{P}(A) = \{\phi, \{2\}, \{3\}, \{4\}, \{5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}, \{2,3,4\}, \{2,3,5\}, \{3,4,5\}, \{2,4,5\}, \{2,3,4,5\}\}$

(10) **Index set :-** $\forall t \in T$ there exist set

for ex:- $T = \{a,b,c\}$ then there are three corresponding set C_a, C_b, C_c, similarly if $T = \{1,2,..n\}$ then they are n corresponding set $C_1, C_2, \ldots C_n$

(11) **Universal set :-**

Operations On set :-

1. **Union :-** Let A and B be two sets the union of A & B is the set of all those elements which are in A and B or in A & B both it is denoted by '$A \cup B$' and read as A union B

ex :- (1) $A = \{2,4,6,8\}$, $B = \{12,13,14,15\}$

$A \cup B = \{2,4,6,8,12,13,14,15\}$

(2) if $A = \{x : 1 < x < 4, x \in I\}$, $B = \{6 < x < 10, x \in I\}$ then $A \cup B = ?$

$\Rightarrow A = \{2,3\}$, $B = \{7,8,9\}$

$\Rightarrow A \cup B = \{2,3,7,8,9\}$

$\Rightarrow$ **Union of Arbitary collection of sets**

let T be a index set and $\forall t \in T$ then corresponding set A_t then t union of arbitary collection of sets at is denoted by $\bigcup_{t \in T} A_t$ and it is the set of all those elements which are in A_t for same $t \in T$

$\{x : x \in A_t, t \in T\}$

ex:- if $T = \{1,2,3,4\}$ & $A_1 = \{a,b,e,f\}$

$A_2 = \{a,c,d,e,f\}$, $A_3 = \{a,b,c,f,g\}$

$A_4 = \{b,c,d,f\}$

$A_T = \{1,2,3,4,a,b,c,d,e,f,g\}$

Disjoint sets:- Two set A and B are called mutually disjoint if they have no element common or in other words their intersection set is empty set.

ex:- $A = \{a, b, c\}$, $B = \{e, f, g\}$

then $A \cap B = \phi$ hence A & B are disjoint set

Difference of two sets

Let A and B be two sets the diff. of A and B is the set of element which belongs to A but doesn't belong to B

The diff. from B to A is denoted by $A - B$ or $A \sim B$, and read as A difference B or A minus B.

ex:- $A - B = \{x : x \in A \text{ and } x \notin B\}$

$B - A = \{x : x \in B \text{ and } x \notin A\}$

from the definition we clearly have.

(a) $A - B \subseteq A$ (b) $B - A \subseteq B$

(c) $(A-B) \cap (B-A) = \phi$

Q1) Suppose set A contains $\{1, 2, 3, 4, 5, 6\}$ $B = \{1, 3, 5, 7\}$ then $B - A = ?$

Sol^n $B - A = \{7\}$

Symmetric diff. b/w sets

If A and B are two sets then their symmetric difference is denoted by $A \Delta B$ and it is defined as

(1) $(A-B) \cup (B-A)$

(2) $(A \cup B) - (A \cap B)$

NOTE:- $A \Delta A = \phi$

$A \Delta \phi = A$

Compliment of set:- Let A be any set and U be a universal set the set $U - A$ is called the Compliment set of A and it is denoted by A' or A^c or $C(A)$

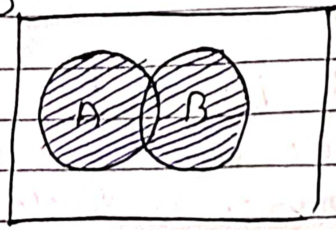
In symbolic notation A' or $A^c = \{x : x \in U \text{ and } x \notin A\}$ if $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7, 10\}$ then $A' = ?$

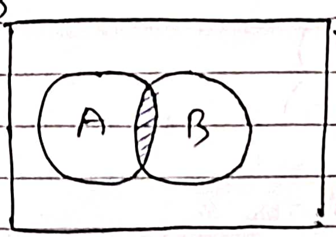
$A' = \{2, 4, 6, 8, 9\}$

Venn diagram

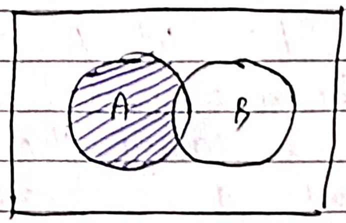
① $A \cup B$



② $A \cap B$



③ $A - B$



Principle of Inclusion & Exclusion

If A and B are ~~infinite~~ finite set then

① $|A \cup B| = |A| + |B| - |A \cap B|$

Q1) At IIPS there are 28 students in ~~an~~ Discrete mathematics class 30 students in Java 8 students in both class How many student ~~in~~ are either in DM or Java class

Sol^n DM = 28, Java = 30, Both = 8

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B| = 28 + 30 - 8$

$|A \cup B| = 50$

② $|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$

Q2) A shop of computer has 8 computers having the following specification

Computer	let A_1 floating pt ~~with~~ arithmatic unit	A_2 Magnetic disk	A_3 Graphical display
I	Y	Y	N
II	Y	Y	Y
III	N	N	N
IV	N	Y	Y
V	Y	N	Y
VI	N	Y	Y
VII	N	N	N
VIII	N	Y	N

Find the no. of computer hardware one or more of three kind of hardware?

Date: / / Page no:

Soln $A_1 = 3$, $A_2 = 5$, $A_3 = 4$

$A_1 \cap A_2 = 2$, $A_1 \cap A_3 = 2$, $A_2 \cap A_3 = \cancel{4}3$

$A_1 \cap A_2 \cap A_3 = 1$

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

$$= 3 + 5 + 4 - 2 - 2 - \cancel{4}3 + 1$$

$$= 6$$

Q3) In a group of athelete Team in a college 21 are in basketball team 26 in hockey and 29 in football Team. If ~~14~~ 14 play Basketball and hockey, ~~15~~ 15 play hockey and football, 8 play all games, 12 play Basketball and football, find total players

Soln $B = 21$, $H = 26$, $f = 29$

$B \cap H = 14$, $H \cap f = 15$, $B \cap f \cap H = 8$

$B \cap f = 12$

$$(B \cup f \cup H) = 21 + 26 + 29 - 14 - 15 - 12 + 8$$

$$= 82 - 41$$

$$= 43$$

Date: / / Page no:

Q4) In High school there are 55 students in either algebra, biology or chem class 28 student in algebra class, 30 student in bio, 24 student in chem class 8 student in both algebra and bio class, 16 student in both bio & chem and 5 student in both algebra & chem

Soln

$$55 = 28 + 30 + 24 - 8 - 16 - 5$$

$$= 82 - 29 + (A \cap B \cap C)$$

$$55 = 53 + (A \cap B \cap C)$$

$$(A \cap B \cap C) = 2$$

Ordered pair & Cartesian Product

An ordered pair consist of two objects in a given fixed order note that an order pair is not set consist two element. the ordering of the two objects is imp. the two objects need not to be distinct we can denote ordered pair by (x, y)

⇒ Cartesian Product

Let A and B be any two sets the set of All ordered pairs such that first member of ordered pair is an element of A & the second pair member is of B is called the Cartesian product of A & B and is written as A×B

$A \times B = \{(x,y) : x \in A \text{ and } y \in B\}$
$B \times A = \{(x,y) : x \in B \text{ and } y \in A\}$

Q) If $A = (4,5)$ and $B = (1,2,3)$ then what is $A\times B$, $B\times A$, $A\times A$, $B\times B$

Sol^n

$\Rightarrow A\times B = \{(4,1), (4,2), (4,3), (5,1), (5,2), (5,3)\}$

$\Rightarrow B\times A = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

$\Rightarrow A\times A = \{(4,5), (4,4), (5,4), (5,5)\}$

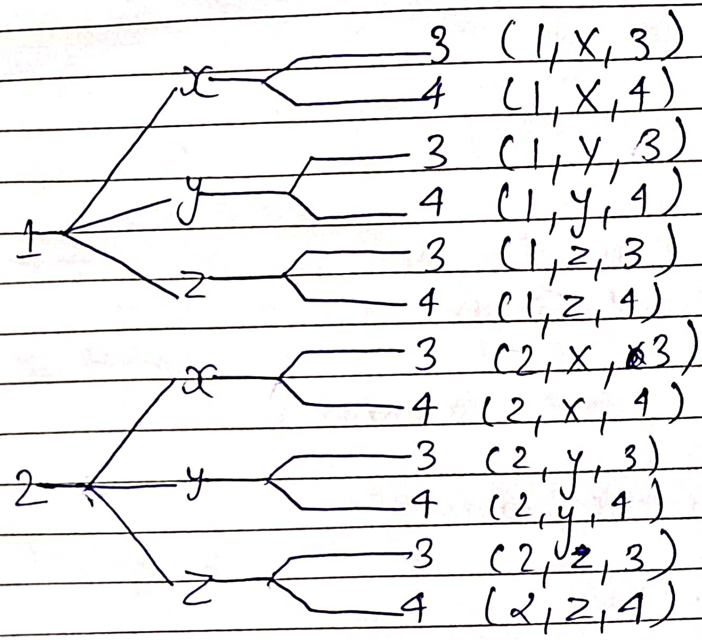
$\Rightarrow B\times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

Q) $A = \{1,2\}$, $B = \{x,y,z\}$, $C = \{3,4\}$ find $A\times B\times C$ and $n(A\times B\times C)$.

Sol^n $A\times B\times$ $\{(1,x), (1,y), (1,z), (2,x), (2,y), (2,z)\}$

A×1

A×B×C consists of all ordered triplet (a,b,c) where $a \in A$, $b \in B$, $c \in C$. These elements of A×B×C can be systematically obtained by a so it is called tree diagram



The element of A×B×C are 12 ordered triplets to right side of tree diagram

$n(A) = 2$
$n(B) = 3$
$n(C) = 3$
$n(A\times B\times C) = 12$
$n(A).n(B).n(C)$

Q) Let $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, d\}$, construct a tree diagram of $A \times B \times C$

Relation

In order to express a relation from set A to set B we always need a statement which connects the element of A with the elements of B

Suppose $A = \{1, 3, 5, 9\}$
$B = \{0, 2, 4, 8\}$

Now suppose a relation from set A to set B is expressed by the statement is less than

$R = \{(1, 2), (1, 4), (1, 8), (3, 4), (3, 8), (5, 8)\}$

Let A and B be two sets a relation from A to B is the subset of $A \times B$. ($R \subseteq A \times B$) and it is denoted by 'R', R is a relation from A to B $\Rightarrow R \subseteq A \times B$

$R = \{(x, y) : xRy$ where $x \in A$ and $y \in B\}$

$\Rightarrow$ Total No. of ~~destrak~~ relation

If the set A has m element and the set B has n element then the total no. of relation from set A to set B is $\underline{2^{m \times n}}$ since $A \times B$ has $m \times n$ elements in all.

$\Rightarrow$ Domain & range of a relation

Suppose R is relation from set A to B i.e $R \subseteq A \times B$. The domain

Date: / / Page no:

of R is the set of all first element of the ordered pair which belongs to R and the Range of R is the set of all second element of the ordered pair.

→ Domain of R = set of all first elements of the ordered pair which belongs to R. ~~such~~ = $\{x : x \in A$ and $(x, y) \in R$ for some $y \in B\}$

→ Range of R = set of all second elements of the ordered pair which belongs to R = $\{y : y \in B$ and $(x, y) \in R$ for some $x \in A\}$

⇒ Inverse Relation

If R is a relation from set A to B then the inverse of R is a relation from B to A and it is denoted by R^{-1}

$$R^{-1} = \{y, x : (x, y) \in R, x \in A, y \in B\}$$

Thus to find R^{-1} we write in reverse order all ordered pairs belonging to R

Range of $R^{-1} \rightleftarrows$ Domain of R

Date: / / Page no:

⇒ The composition relation of two relations

Let A, B and C be three sets. Suppose R is a relation from set A to B and S is the relation from set B to C. The composition relation of two relations R & S is the relation from set A to C and it is denoted by SoR and defined as SoR = $\{(a, c) :$ there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S\}$ where $a \in A$, $c \in C$. Hence $(a, b) \in R$, $(b, c) \in S \Rightarrow (a, c) \in SoR$

⇒ Binary Relation OR Relation on a Set

Suppose A is a non empty set a relation R in set A is the subset of A×A clearly both the co-ordinates of ordered pair in R are the elements of the set A the relation R in the set A is called a Binary relationship

☆ Properties of Binary Relation

① Reflexive :- If R is a relation in the set A then R is called Reflexive

Date: / / Page no:

relation if every element of A is related to itself

$(a,a) \in R, \forall a \in A$

$aRa \ \forall a \in A$

$R \subseteq A \times A$

if $A = \{1,2,3\}$
$R = \{(1,1),(2,2),(3,3)\}$

⇒ Symmetric Relation 24/01/2023

If R is a relation in the set A then R is called symmetric relation if a is R related to b

i.e $(a,b) \in R \Rightarrow (b,a) \in R$

i.e $aRb \Rightarrow bRa$

where $a,b \in A$

ex:- If $A = \{2,4,5,6\}$

symmetric (a) $R_1 = \{(2,4),(4,2),(4,5),(5,4)\}$

Not (b) $R_2 = \{(2,4),(2,6),(6,2),(5,4),(4,5)\}$

⇒ Antisymmetric Relation:- If R is a relation in the set A then R is called Antisymmetric if $(a,b) \in R$ and $(b,a) \in R$ ⇒ $a=b$ where $a,b \in A$

ex:- in a set of natural no. the relation "a divides b" is antisymmetric since "a divides b" and "b divides a" is possible only when $a=b$ it means if the given relation be denoted by R then $(a,b) \in R$ and $(b,a) \in R \Rightarrow a=b$

Date: / / Page no:

⇒ Transitive relation:- If R is a relation in the set A the R is called transitive relation if a is R related to B and b is R related to C then there must be a is releated to C

ex:- $A = \{1,2,3\}$

then $R = \{(1,2),(2,3),(1,3)\}$

⇒ $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

aRb and $bRc \Rightarrow aRc$

$a,b,c \in A$

⇒ Identity relation:- If R is a relation in set A then R is called identity relation if a is R related to b or

$aRb \Rightarrow a=b \ \forall a,b \in A$

ex:- If $A = \{2,4,6\}$

$R = \{(2,2),(4,4),(6,6)\}$

Note:- every identity relationship is reflexive but the converse is not true

⇒ Equivalence relation:- Suppose R is the relation in set A then R is called equivalence relation if

(1) R is reflexive $(a,a) \in R \ \forall a \in A$

(2) R is symmetric $(a,b) \in R \Rightarrow (b,a) \in R$

Date: / / Page no:

where $a,b \in A$

③ R is transitive $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ also where $a,b,c \in A$

Ques1) If $R_1 = \{(a,b) \mid a-b$ is an integer$\}$
$R_2 = \{(a,b) \mid a-b$ is divisible by $3\}$
$R_3 = \{(a,b) \mid a-b$ is an odd no.$\}$
$R_4 = \{(a,b) \mid a-b$ is an even no.$\}$

Solⁿ ① $R_1 = \{(a,b) \mid a-b$ is an integer$\}$

i) Reflexive :- let 2 belongs to the set then, $2-2=0$ it is an integer

Hence it is a reflexive relation

ii) Symmetric :- let $1, 2 \in$ set
then, $1-2 = -1 = -1$ | $2-1 = 1 = 1$

Both are integers Hence it is symmetric

iii) transitive :- let $1,2,3 \in$ set
then $(1,2) \in R$ and $(2,3) \in R$ then $(1,3)$ also $\in$ to R
$\Rightarrow 1-2 = -1$ | $2-3 = -1$ | $1-3 = -2$
$\Rightarrow$ all three are integers
Hence it is transitive and it is equivalence.

② $R_2 = \{(a,b) \mid a-b$ is divisible by $3\}$
i) Reflexive :- let $1 \in$ set
$1-1=0 \Rightarrow 0$ is divisible by 3
$\Rightarrow$ Hence it is reflexive

ii) symmetric :- let 3 and 6 $\in$ set
then $(3,6) \in R \Rightarrow (b,a) \in R$
$\Rightarrow 3-6 = -3$ | $\Rightarrow 6-3 = 3$
$\Rightarrow -3$ is divisible by 3 | $\Rightarrow 3$ is divisible by 3

Hence it is symmetric relation

iii) transitive :- let 3, 6 and 9 $\in$ set then
$\Rightarrow (3,6) \in R$ and $(6,9) \in R$ then $(3,9)$ also $\in$ set
$\Rightarrow 3-6 = -3$ | $6-9 = -3$ | $3-9 = -6$
$\Rightarrow -3$ is divisible by 3 | $\Rightarrow -3$ is divisible by 3 | -6 is divisible by 3

$\Rightarrow$ Hence it is transitive relation also it is equivalence relation

③ $R_3 = \{(a,b) \mid a-b$ is an odd no.$\}$
i) Reflexive :- let $1 \in$ set
$\Rightarrow 1-1=0$, 0 is not an odd no
Hence it is not reflexive

ii) Symmetric :- let -3 and $-9 \in$ set
$\Rightarrow -3+9 = 6$
$\Rightarrow$ Hence it is symmetric

(iii) transitive :- let 1, 2, and 3 $\in$ set
$\Rightarrow (1,2) \in R$ and $(2,3) \in R$ then $(1,3) \in$ set
$\Rightarrow 1-2 = -1$ | $2-3 = -1$ | $1-3 = -2$

Date: / / Page no:

Hence it is not transitive

$\Rightarrow$ It is not equivalence relation also

④ $R_4 = \{(a,b) \mid a-b$ is an even No.$\}$

Soln i) reflexive :- let $1 \in$ set

$\Rightarrow 1-1=0$, 0 is an even no.

Hence it is reflexive

ii) Symmetric :- let $1, 3 \in$ set

$\Rightarrow (1,3) \in \cancel{set}\,R \Rightarrow (3,1) \in \cancel{set}\,R$

$1-3=-2$ | $3-1=2$

-2 is an even no | $\Rightarrow 2$ is an even no.

Hence it is symmetric

iii) transitive :- let 1, 3, and 5 $\in$ set

$\Rightarrow (1,3) \in \cancel{set}\,R$ and $(3,5) \in \cancel{set}\,R$ then $(1,5)$ also $\in \cancel{set}\,R$

$\Rightarrow 1-3=-2$ | $3-5=-2$ | $1-5=-4$

-2 is an even no | $\Rightarrow -2$ is even | $\Rightarrow -4$ is even

Hence it is transitive and it is also equivalence.

Equivalence Class :-

$[x] = \{y \mid y \in A$ and $(x,y) \in R\}$

$A = \{1, 2, 3, 4, 5\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (4,5), (5,4)\}$

Date: / / Page no:

$[1] = \{1, 2\}$

$[2] = \{2, 1\}$ and $\{1, 2\}$ are same

write any one of them

$[3] = \{3\}$

$[4] = \{4, 5\}$

$[5] = \{5, 4\}$

$P_1 = \{1, 2\}$, $P_2 = \{3\}$, $P_3 = \{4, 5\}$

$\Rightarrow P_1 \cup P_2 \cup P_3 = A$

$\Rightarrow P_1 \cap P_2 \cap P_3 = \{\phi\}$

25/01/2023

(Imp) # Representing Relation using matrices

A relation b/w finite sets can be represented using a 'zero-one' matrix. Suppose that R is the relation from $A = \{a_1, \ldots a_n\}$ to $B = \{b_1, \ldots b_n\}$ the relation R can be represented by the matrix $M_R = [M_{ij}]$ where
$$M_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{if } (a_i, b_j) \notin R \end{cases}$$

Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$ let R be the relation from set A to set B containing (a,b) if $a \in A$, $b \in B$ and $a > b$ then what is the matrix representation of 'R'

Soln $R = \{(2,1), (3,1), (3,2)\}$

$$M_R = \begin{array}{c|cc} & b_1 & b_2 \\ \hline a_1 & 0 & 0 \\ a_2 & 1 & 0 \\ a_3 & 1 & 1 \end{array}$$

~~relation~~ $R = \{(x,y) : x, y \in N$ and ~~x+y~~ $\}$

Q1) Let I be the set of all integers. Let's define the relation R in set I such that xRy if $x-y$ is divided by 5, $x \in I$ and $y \in I$

i.e. $R = \{(x,y) : x \in I, y \in I$ and $x-y$ divisible by $5\}$

Sol^n (i) Reflexive :- let $1 \in I$

$\Rightarrow 1 - 1 = 0$

$\Rightarrow 0$ is divisible by 5

Hence it is reflexive

(ii) Symmetric :- let $10, 5 \in I$

$\Rightarrow 5 - 10 = -5 \mid 10 - 5 = 5$.

$\Rightarrow -5$ and 5 is divisible by 5.

Hence it is symmetric

(iii) Transitive :- let $5, 10, 15 \in I$

$\Rightarrow (5,10) \in R$ and $(10,15) \in R$

$\Rightarrow (5,15) \in R$

$\Rightarrow 5 - 10 = -5 \mid 10 - 15 = -5 \mid 5 - 15 = -10$

Hence it is transitive

for exam :-

Sol^n (i) Reflexive :- for each $x \in I$ we have $x - x = 0$ is divisible by 5

Thus $\forall x \in I$ we get xRx

Therefore R is reflexive

(ii) Symmetric :- let $x, y \in I$ we have $x-y$ is divisible by 5

$\Rightarrow -(x-y)$ is divisible by 5

$\Rightarrow (y-x)$ is divisible by 5

Thus $xRy \Rightarrow yRx$

Therefore R is symmetric

(iii) Transitive :- let $x, y, z \in I$

we have xRy, yRz

$\Rightarrow x-y$ divisible by 5

$\Rightarrow y-z$ divisible by 5

$\therefore x-y+y-z = (x-z)$ is also divisible by 5

$yRz = xRz$ is transitive

Since R is reflexive, symmetric & transitive $\Rightarrow R$ is equivalence

Q) In a set of L of all straight line in a plane. Find which of the relation are equivalence relation

Date: / / Page no:

$R_1 = \{(x,y) : x, y \in L$

Sol^n (i) reflexive for any $x \in L$ we have $x \parallel$ to itself

$\Rightarrow (x,x) \in R_1 \quad xRx \Rightarrow R_1$

Hence R_1 is reflexive

(ii) Symmetric:- for any $x \in L$ and $y \in L$ we have $x \parallel y$

$\Rightarrow y \parallel x$

$\Rightarrow xRy \Rightarrow R_1$

Hence R_1 is symmetric

Date: / / Page no:

Q) Let R be ~~an~~ a relation $N \times N$ where N is a set of +ve integer defined by $(a,b) R (c,d) \Rightarrow a+d = b+c$ and $a,b,c,d \in N$ then prove that R is an equivalence relation or not

Sol^n

Partial ordered relation

A relation R on a set A is called Partial ordered relation if R is

1. reflexive i.e aRa $\forall a \in A$
2. R is anti symmetric i.e aRb, bRa $\Rightarrow a=b$ for each $a,b \in A$
3. R is transitive i.e. aRb, bRc $\Rightarrow aRc$ for each $a,b,c \in A$

Function :- A function is a special kind of relation. In many instances we assigned each element of first set to a particular element of second set for example suppose that each student in discrete mathematic's class is assigned a letter from the $\{A, B, C, D, F\}$ and suppose that Grad are A = Outstanding, B = excellent

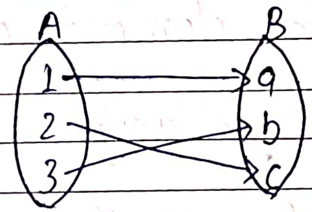
C = Very Good, D = Good

F = Fail

The concept of function is extremely important in Discrete mathematic The functions are used in definition such as Discrete structure at Sequence & strings

Suppose that to each element of A there is assigned a unique element of B. The collection of such assignment is called a function or Mapping or Map from A to B if there are two set We donate a function f from A to B by $f: A \to B$ and read as f of A onto B.

Where the set A is a domain of f and the set B is the Co-domain of f we write $f(a)$, and read as f of a for the element of B that f assigns to $a \in A$ and it is called the value of f at 'a' or the image of 'a' under f.



$f(1) = a$
$f(2) = c$
$f(3) = b$

$\Rightarrow$ Equality of function :- Two function $f: A \to B$ and $g: A \to B$ are defined to be equal if $f=g$ if $f(a) = g(a)$ $\forall a \in A$. The negation of $f = g$ is $f \neq g$ and is the statement there exists an $a \in A$ for which $f(a) \neq g(a)$.

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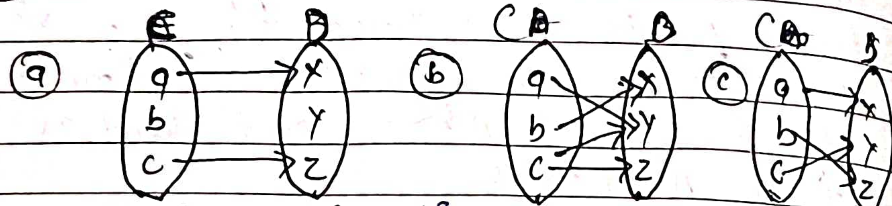
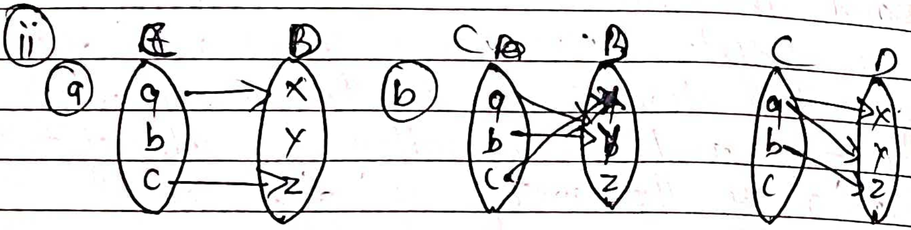
Ques1) state whether or not each diagram below defined as function from $C A = \{a, b, c\}$ $B(x, y, z)$

(i)



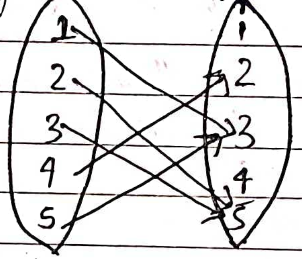
Soln (c) is a function

(ii)



Soln No there no element in D
b∈A
element of C is assigned a unique element of D
No two element x and y a∈C

Ques2) Consider the set $A = \{1, 2, 3, 4, 5\}$ the function $f: A \times A$ defined by



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(1)
(1) find the images of each element of A
(2) and the image $f(A)$ of the function f

Soln (1) $f(1) = 3$, $f(2) = 5$, $f(3) = 5$, $f(4) = 2$
$f(5) = 3$
(2) The arrow indicates the images of an element.

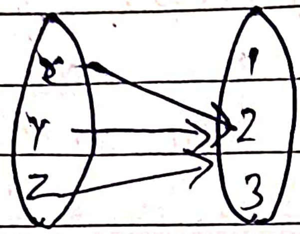
(2) The image $f(A)$ of f consist all the image value, only 2, 3, 5 appears as a image of any element
Thus $f(A) = \{2, 3, 5\}$

(ii) (1) find $f(S)$ where $S\{1, 3, 5\}$
(2) $f^{-1}(T)$ where $T = \{1, 2\}$
(3) $f^{-1}(3)$

Soln (1) $f(1) = 3$, $f(2) = 5$, $f(5) = 3$
$\{3, 3, 5\} = \{3, 5\}$
(2) $f^{-1}(2) = 4$

(3) $f^{-1}(3) = \{1, 5\}$

element this can be represented by a diagram where all images



$f(x) = 2$
$f(y) = 2$
$f(z) = 2$

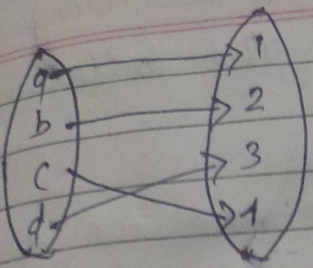
Types of Mapping

→ Injective or one one mapping

A mapping ~~f(x) ist~~ f of x into y is said to be injective or one one mapping if distinct element of x have distinct images in y it is called injective or one-one mapping

ex:- $f: x \to y$ is a injective (one-one) mapping if and only if
$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

ex:- if $X = \{a, b, c, d\}$ $Y = \{1, 2, 3, 4\}$ and $f\ x \to y$ defined as
$f(a) = 1$ $f(b) = 2$, $f(c) = 4$
$f(d) = 3$



hence it is injective or one-one

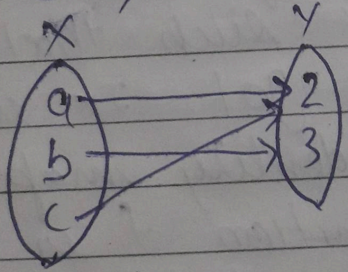
Surjective or Onto mapping :-

If the mapping f of x into y is such that every element of y has the image of atleast one element of x then the mapping is called surjective or onto

In other words

$f : X \rightarrow y$ is onto y if given $y \in Y$ there exist an element $x \in X$ such that $y = f(x)$

ex:- if $X = \{a, b, c\}$, $y = \{2, 3\}$ and if $f : x \rightarrow y$ is defined by $f(a) = 2$, $f(b) = 3$, $f(c) = 2$



It is surjective

Bijective or One-One onto mapping

A mapping which is one-one as well as onto is called bijective or one-one onto mapping. To determine whether the mapping is bijective we have to follow the following procedure:-

- To show if f is one-one we must show that $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- To show if f is onto y we must show that for each $y \in Y$ there exist an element $x \in X$ such that $f(x) = y$

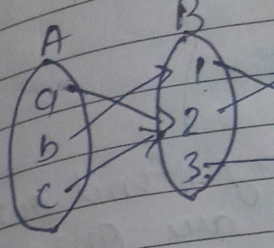
ex:- If $x = \{a, b, c, d\}$ $y = \{1, 2, 3, 4\}$
and $f(a) = 1$, $f(b) = 2$, $f(c) = 4$
$f(d) = 3$
It is one-one and onto i.e Bijective

Invertible function:-

function f of x into y is said to be invertible if there exist a function $g: y \rightarrow x$ such that
$fog = I_y$ and $gof = I_x$ where I_x and I_y are identity map in such case, a function f is called inverse function and it is denoted by 'f^{-1}'

Let the function $f: A \rightarrow B$, $g: B \rightarrow C$ and $h: C \rightarrow D$ be defined by below diagram Determine which of the function are

① one-one ② On



Soln ① ⇒ g: f: A
because
⇒ g: B→C
every
emage
⇒ h: C→D
h(x)

② f: A→B
3 does
⇒ g: B→
z does
⇒ h: C→
of D h

③ f: A→B

following

we must
$) \Rightarrow x_1 = x_2$
we must
$\in Y$ there
such that

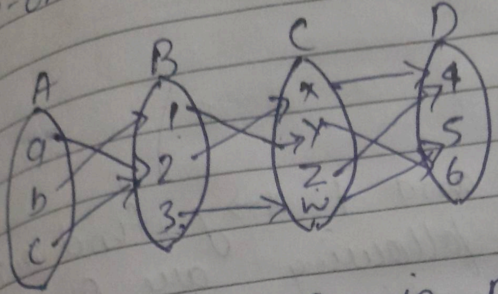
$\{1, 2, 3, 4\}$
, $f(c) = 4$

to i.e ~~Surjective~~ Bijective

is said
here exist a
that
I_x where I_x
map in
f is called
t is denoted

$\rightarrow B$, $g: B \rightarrow C$
ned by below
hich of the

① One-one ② Onto ③ Invertible



Soln ① ⇒ g: $f: A \rightarrow B$ is not one-one because $f(a) = 2$ & $f(c) = 2$

⇒ $g: B \rightarrow C$ is one-one because every element of B has a unique image

⇒ $h: C \rightarrow D$ is not one-one because $h(x) = 4$ and $h(z) = 4$

② $f: A \rightarrow B$ is not onto because 3 doesn't have any image in A

⇒ $g: B \rightarrow C$ is not onto because z doesn't have any image in B

⇒ $h: C \rightarrow D$ is onto as every element of D has atleast one image in C

③ $f: A \rightarrow B$

Ques1) If $X = \{0, \infty\}$ and $f: X \to y$ find out which of the following are onto (surjective) mapping

(i) $f(x) = x^2$ (ii) $f(x) = x^3+1$

(iii) $f(X) = x+2$

Soln We have $f(X)$ when $0 \in X$

(1) $\Rightarrow f(0) = 0^2 = 0 \in X$

when $\infty \in X$

$f(\infty) = \infty^2 \Rightarrow \infty \in X$

Hence it is onto

(2) $f(X) = X^3 + 1$

$\Rightarrow f(0) = 0 + 1 \Rightarrow 1 \notin X$

$\Rightarrow f(\infty) = \infty + 1 \Rightarrow \infty \in X$

Since $1 \notin X$ so it is not onto

(3) $f(x) = x+2$

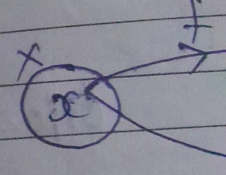
$\Rightarrow f(0) = 0+2 \Rightarrow 2 \notin X$

$\Rightarrow f(\infty) = \infty + 2 \Rightarrow \infty \in X$

Since $2 \notin X$ so it is not onto

Composite of fun
Let there be
suppose two
defined by $y =$
$y \in Y$ and g
$z = g(y)$

f and g is
by gof
$f: X \to y\ z$
$gof: X \to z$
$\boxed{(gof)(x)}$



ex:- Let f a
set of
integers
$g(x) = 3x$
what is
what is
Soln $(fog)x$

$2(3x+2)$
$(gof)x$

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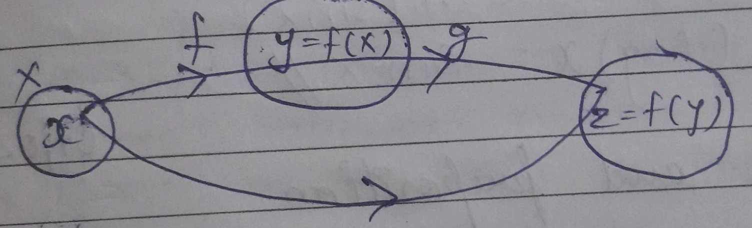
Composite of function

Let there be 3 sets X, Y & Z
suppose two function $f: X \to Y$ is
defined by $y = f(x)$ where $x \in X$ &
$y \in Y$ and $g: Y \to Z$ is defined by
$z = g(y)$ where $y \in Y$ and $z \in Z$

f and g is composite mapping denoted
by $g \circ f$ and if a function
$f: X \to Z$ dentoded by
$g \circ f: X \to Z$ where

$$\boxed{(g \circ f)(x) = g[f(x)] \ \forall x \in X}$$ *



ex:- Let f and g be the function from set of integers to the set of integers defined by $f(x) = 2x+3$ and $g(x) = 3x+2$
what is the composition of f and g and
what is the composition of g and f

Soln $(f \circ g)x = f[g(x)] = f(3x+2)$

$2(3x+2)+3 = 6x+4+3 = 6x+7$ —— (1)

$(g \circ f)x = g[f(x)] = g[2x+3]$

Date: / / Page no:

$3(2x+3)+<6x+11 \longleftarrow (2)$

Ques1) If $f: R \to R$ defined by $f(x) = x^2$ $\forall x \in R$ and $g: R \to R$ is defined by $g(x) = \sin x$ $\forall x \in R$ then find gof and fog and also show whether $gof = fog$ or $gof \neq fog$

Soln

$\Rightarrow (gof)(x) = g[f(x)]$

$(gof)(x) = g[(\sin x)^2]$ //

$\Rightarrow (fog)x = f[g(x)]$

$= f[\sin x]$

$\Rightarrow (fog)x = \sin^2 x$ //

07/02/2023

Logic and preposition

⇒ **Sentence** :- A group of words which may not having its value true or false

⇒ **statement** :- It is a special kind of sentence which is defined as its value true or false

⇒ **Logical connections** :- They are the words or symbols used to combine two

Sentences to
or a comp

	Connective words
1	not
2	and
3	or
4	if.... then
5	iff or if & only if

(1) NEGATION:
Truth
Inpu
0 (F
1 (T

x –

(2) Conjuction
statemen
~~word~~ are
word 'A
represent
P and
AND

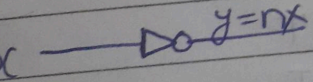
Sentences to form a compound sentence or a compound statement.

	Connective words	Name of connectives	Symbol	Rank
1	not	Daniel/Negation	~ or ¬	1
2	and	conjunction	$\wedge$	2
3	or	disjunction	$\vee$	3
4	if...then	conditional	$\Rightarrow$ or $\rightarrow$	4
5	iff or if & only if	Bi-conditional	$\Leftrightarrow$ or $\leftrightarrow$	5

① NEGATION :-

Truth Table $y = \neg x$

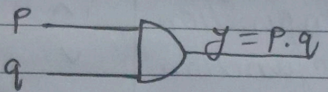
Input	Output
0 (F)	1 (T)
1 (T)	0 (F)



② Conjuction :- suppose there are two statements p and q when two statement ~~word~~ are combined while using the word 'AND' then we get a new statement represented as : $p \wedge q$ and read as p and q.

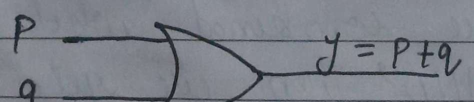
AND → p.q

Input		Output
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F



(3) Disjunction :- When two statement p and q are join by using the word 'or' then we get a new statement represented by $p \vee q$ and read as p or q its value is true if any one of the statement is true

Input		Output
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F



OR → p+q

(4) Conditional :- ...
-ent then the ...
p then q is ...
ment and is ...
Its value is ...
& q is false ...

Input	
p	q
T	T
T	F
F	T
F	F

(6) Bi- Condition...
statements ...
the type ...
bi- condi...
value is ...
are either ...

Input	
p	q
T	T
T	F
F	
F	

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(5) **Conditional :-** If P & q are two statem-ent then the statement of type if p then q is called conditional state-ment and is represented by $P \to q$. Its value is false if P is true & q is false

Input		Output
P	q	$P \to q$
T	T	T
T	F	F
F	T	T
F	F	T

(6) **Bi-Conditional :-** If P and q are two statements then the statement of the type $P \to q$ and $q \to P$ is called bi-conditional statement and its value is true when both P & q are either true or false

Input		Output		
P	q	$P \to q$	$q \to P$	$P \to q$ and $q \to P$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Tautology :- It is a preposition which is true for all truth various of its sub preposition on components.

It is also called as logically valid or logically true statements. Any compound statement is said to be a tautology if it has all the truth values true

Contradiction :- A contradiction is a preposition which is always false for all truth values of its sub preposition or components.

Also called as logically invalid or logically false statement. Any compound statement is said to be a contradiction if it has all the truth values false

Logical equivalence :- Two statements are said to be logically equivalent if truth values of both the statement are always identical.

And it is written as p logically equivalent q $\{P \equiv q\}$

Contengency :-

ex:- if P & q then find weth

$P \wedge q \Rightarrow P$ is

① $(P \wedge q) \Rightarrow (P \vee$

② $(P \Leftrightarrow q) \Leftrightarrow (P \Rightarrow$

③ $(P \Rightarrow q) \wedge (q \Leftarrow$

④

Ans

P	q
T	T
T	F
F	T
F	F

Yes tautolog

②

P	q
T	T
T	F
F	T
F	F

Yes tautolog

③

P	q
T	T
T	F
F	T
F	F

position
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as logically
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p logically

Contengency:-

ex:- if p & q are two statements
then find wether :-
(1) $p \wedge q \Rightarrow p$ is tautology or not
(2) $(p \wedge q) \Rightarrow (p \vee q)$
(3) $(p \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \rightarrow p)$
(4) $(p \Rightarrow q) \wedge (q \leftrightarrow r) \longrightarrow (p \rightarrow r)$

Ans

p	q	$p \wedge q$	$p \wedge q \Rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Yes. tautology

(2)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

yes tautology

(3)

p	q	$p \Leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \Rightarrow q) \Leftrightarrow (p \rightarrow q)$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

Output

T

T

F

T   No tautology

(4)

Ques) PT $(P \vee q) \wedge (\sim P) \wedge (\sim q)$ is contradiction or not

Prove that $P \Rightarrow (q \Rightarrow r) \equiv (P \wedge q) \Rightarrow r$

whether the above preposition logical equivalance or not.

p	q	$\sim p$	$\sim q$	$p \vee q$	$(p \vee q) \wedge (\sim p)$
T	T	F	F	T	F
T	F	F	T	T	F
F	T	T	F	T	T
F	F	T	T	F	F

$(P \vee q) \wedge (\sim P) \wedge ($

F

F

F

F

yes it i

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Date: / / Page no:

$(p \vee q) \wedge (\sim p) \wedge (\sim q)$

F

F

F

F

yes it is a contradiction

p	q	r	$(q \Rightarrow r)$	$p \Rightarrow (q \Rightarrow r)$	$(p \wedge q)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	F
T	F	F	F	F	F
F	T	T	T	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	F	F	F

Boolean Algebra

Let ~~be~~ B is the set (non empty) with element {a, b, c} which is defined on two operation '+' and '.' then the algebraic structure (R+P) or (B, is said to be Boolean algebra if B satisfied the following property or laws or preposition

→ closure law :-

if $a, b \in B \Rightarrow a+b \in B$

$\Rightarrow ab \in B$

→ Commutative law :-

$a+b = b+a$

$a \cdot b = b \cdot a \quad \forall\, a, b \in B$

→ Distributive law :-

$a+(b \cdot c) = (a+b)(a+c)$

$\forall\, a, b, c \in B$

$a \cdot (b+c) = ab+ac \quad \forall\, a, b, c \in B$

→ Identity law :- there should be identity elements on both side of operation identity element of + = 0

$a+I = a \quad I = 0$

for multiplic

$=1 \quad a \cdot I = a$

0 and 1 sd

w.r.to oper

$0+a = a+$

$a \cdot 1 = 1 \cdot a$

→ Inverse law

corresponds

should ha

if $a \in B$

then $a+a$

⇒ Properties of

→ Idempotent

- $a+a = a$

$\Rightarrow a+a = a$

$\Rightarrow (a+a) \cdot$

$\Rightarrow (a+a) \cdot ($

$\Rightarrow a+aa$

$\Rightarrow a+0$

$\Rightarrow a$

- $a \cdot a = a$

$\Rightarrow (a \cdot a)+$

$\Rightarrow a \cdot a+$

$\Rightarrow a(a+a$

for multiplication identify element is $=1$ $\quad a.I = a \quad I=1$

0 and 1 ~~is~~ are identity element w.r.to operator '+' and '.'

$0+a = a+0 = a \quad \forall a \in B$

$a.1 = 1.a = a \quad \forall a \in B$

→ Inverse law :- every element of set B corresponds both the operation should have inverse element if $a \in B$ then a' is inverse of a then $a+a' = 1$ and $a.a' = 0$

⇒ Properties of Boolean algebra

→ Idempotent law :-

- $a+a = a$

$\Rightarrow a+a = a$

$\Rightarrow (a+a).1$ [Identity law]

$\Rightarrow (a+a).(a+a')$ [Inverse law]

$\Rightarrow (a+a).(a+a')$ [Distributive law]

$\Rightarrow a+aa'$ [Identity law]

$\Rightarrow a+0$

$\Rightarrow a$

- $a.a = a$ [Identify law]

$\Rightarrow (a.a)+0$ [Inverse law]

$\Rightarrow a.a+aa'$ [Distributive law]

$\Rightarrow a(a+a')$ [Distributive law]

Date: / / Page no:

$\Rightarrow a.1$ [Identity law]

$\Rightarrow a$

- $a + 1 = 1$
- $a.0 = 0$

→ Absorption law

- $a + a.b = a$
- $a(a+b) = a$

Ques1) In a boolean algebra B prove that Identity elements are complementary to each other it means for $0, 1 \in B$

① $0' = 1$ ② $1' = 0$

① $0' + 0 \Rightarrow 1 + 0$ (By Identity law)

$\Rightarrow 1$// Hence proved

② $1' = 0$

$\Rightarrow 1'.1$ (By Identity law)

$\Rightarrow 0$// Hence proved

Ques2) Simplify following using Boolean algebra

(i) $(a+b)a'b'$ (ii) $abc + a' + b' + c'$

(iii) $(ab' + c')'$ (iv) $[a + a'b][a' + ab]$

(v) $ab + [(a+b').b]'$

Ques3) Let P = it is c
Give a simple
which describe
statements

(i) $\sim P$ (ii) $P \wedge q$ (i

(v) $P \Rightarrow \sim q$ (vi) $q \vee$

(viii) $P \Leftrightarrow \sim q$ (ix)

(xi) $\sim\sim P$ (xii)

Ques4) Let P be "
Let q be "
Write each
form

(a) Ravi is shor

(b) Ravi is Tall

(c) It is not true
not handsom

Ques5) Let P be "
Let q be "
simple ver
each of the

(a) $P \vee q$ (b) $P \wedge$

(e) $\sim(\sim P)$

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Ques3) Let P = it is cold, q = it is raining
Give a simple verb sentences which describe each of the following statements

(i) $\sim P$ (ii) $P \wedge q$ (iii) $P \vee q$ (iv) $q \Leftrightarrow P$
(v) $P \Rightarrow \sim q$ (vi) $q \vee \sim P$ (vii) $\sim P \vee \sim q$
(viii) $P \Leftrightarrow \sim q$ (ix) $\sim\sim q$ (x) $(P \wedge \sim q) \Rightarrow P$
(xi) $\sim\sim P$ (xii) $(P \wedge \sim q) \Rightarrow q$

Ques4) Let P be "Ravi is tall and
Let q be "Ravi is handsome
Write each statement in the symbolic form

(a) Ravi is short or handsome $\sim P \vee q$
(b) Ravi is tall or handsome $P \vee q$
(c) It is not true that Ravi is short or not handsom $\sim\sim P \vee \sim q$

Ques5) Let P be "Ravi speaks Tamil" and
Let q be "Ravi speaks hindi" Give a simple verbal sentence which describe each of the following

(a) $P \vee q$ (b) $P \wedge q$ (c) $P \wedge \sim q$ (d) $\sim P \vee \sim q$
(e) $\sim(\sim P)$

Demougan's law

→ $(a+b)' = a'b'$
→ $(ab)' = a'+b'$

Quantifier :- when variable is a preposition function. Assume a value, the resulting statement becomes a preposition with truth values.

In quantifier we create preposition from a preposition function. It extent to which predict is true on a range of element.

In english words all, some, many, none and few are used as quantification

⇒ There are two types of quantifier

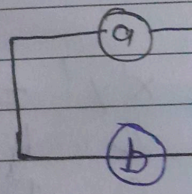
- Extential quantifier or quantification:- which tells us that a predicate is true for every element under the condition
- Universal quantifier or quantification:- which tells us that there is one or more element for predicate is true

The are whic
quantifier is

Algebra of e

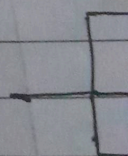
Parallel =
Series =

Ques1) find the
function
following



Soln $f =$
$f_1 = a+$
$f_2 = C$

Ques2)



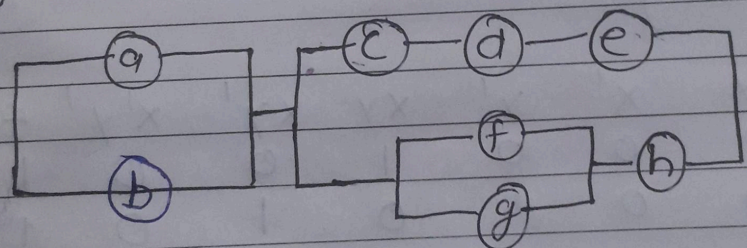
Draw
followin

which
The are which deals with predicates & quantifier is called as Predicate Calculus

Algebra of electric circuit

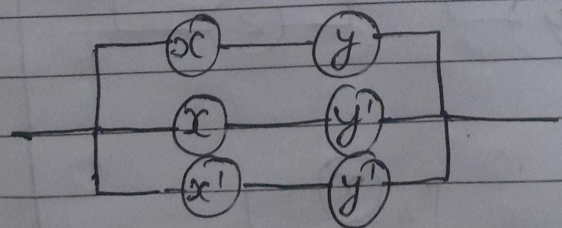
Parallel = o/p $\rightarrow x+y$
Series = o/p $\rightarrow x \cdot y$

Ques1) Find the switching net or switching function or polynomial for the following circuit



Soln
$f = f1 \cdot f2$
$f_1 = a+b$
$f_2 = cde + ((f+g) \cdot h)$

Ques2)



Draw a simple circuit for the following diagram and also verify the equivalent circuit and the circuit with the help of truth table

Soln

$f = f1 . f2 . f3$

$\Rightarrow f1 = x \cdot y$

$\Rightarrow f2 = xy'$ $\Rightarrow f3 = x'y'$

$\Rightarrow f = (x \cdot y) + (x \cdot y') + (x' \cdot y')$

$\Rightarrow x(y+y') + x'y'$ [DL]

$\Rightarrow x + x'y'$ [IL]

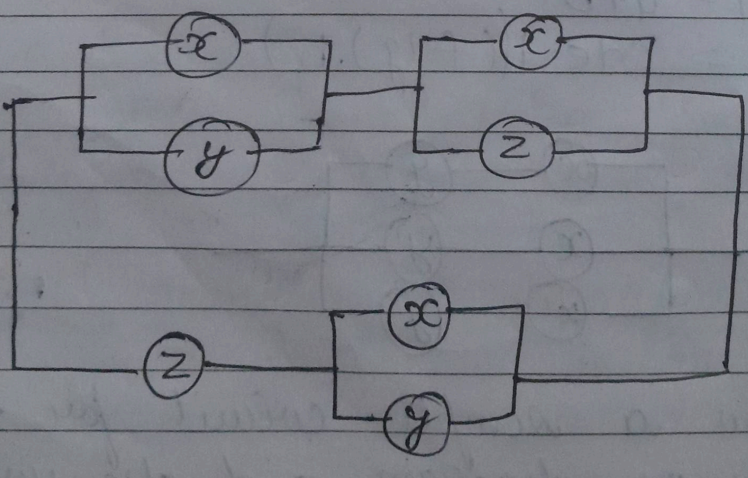
$\Rightarrow (x+x')(x+y')$ [DL]

$\Rightarrow 1 . x+y'$

$\Rightarrow x+y'$

X	Y	X'	Y'	XY	XY'	X'Y'	f	X+Y'
1	1	0	0	1	0	0	1	1
1	0	0	1	0	1	0	1	1
0	1	1	0	0	0	0	0	0
0	0	1	1	0	0	1	1	1

Ques3



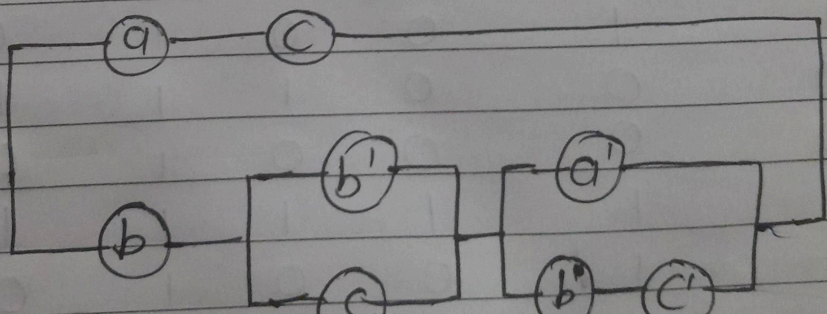
Ques 4)

circuit
table

)

y'	f	x+y'
	1	1
	1	1
	0	0
	1	1

Ques 4)



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Soln $f_1 = ac$

$f_2 = b.(b'+c) \, . [a' + (bc')]$

$f = f_1 + f_2$

$= ac + [b.(b'+c).(a' + (bc'))]$

$= ac + [(bb' + bc).(a' + (bc'))]$

$= (ac + bc).(a' + bc')$

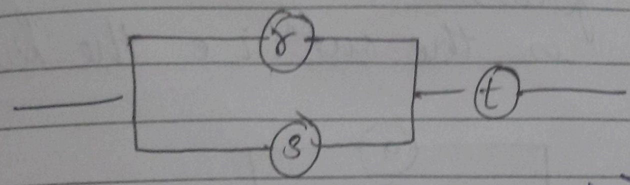
$= c(a+b).(a' + bc')$

a	b	c	a'	b'	c'	f_1	b'+c	b.(b'+c)
1	1	1	0	0	0	1	1	1
1	1	0	0	0	1	0	0	0
1	0	1	0	1	0	1	1	0
1	0	0	0	1	1	0	1	0
0	1	1	1	0	0	0	1	1
0	1	0	1	0	1	0	0	0
0	0	1	1	1	0	0	1	0
0	0	0	1	1	1	0	1	0

bc'	a'+(bc')	f_2	f	a+b	c.(a+b)	Output
0	0	0	1	1	1	0
1	1	0	0	1	0	
0	0	0	1	1	1	
0	0	0	0	1	0	
0	1	1	1	1	1	
1	1	0	0	1	0	
0	1	0	0	0	0	
0	1	0	0	0	0	

Date 27/02/2022 Page no:

Ques1) PT the Boolean function $rt + [s(s'+t)\{r' + (st)\}]$ is replaced by the following net



Soln

$rt + [s(s'+t)\{r' + (st)\}]$

$rt + [ss' + st\{r' + (st)\}]$ [∵ By DL]

$rt + [0 + st\{r' + (st)\}]$ [∵ $aa' = 0$]

$rt + [str' + st]$ [∵ $st \cdot st = st$]

$rt + st[r' + 1]$

~~st~~ $rt + st$ [$a' + 1 = 1$)

$t(r + s)$

Hence proved

$(b'+c)$
1
0
0
0
1
0
0
0

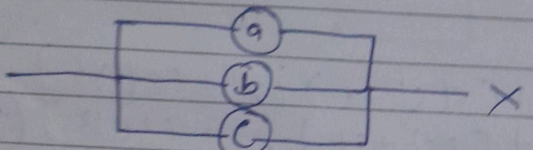
Ques2) Draw a simplified circuit of an expression $xy'z + (z+y)x'$

b) Output
0

~~Ques~~ Design of Synthesis Control System

Ex:- The bulb inside a car of two doors is on when one of these door is open and it is also on when a switch of deskboard is pressed draw a diagram for the control path

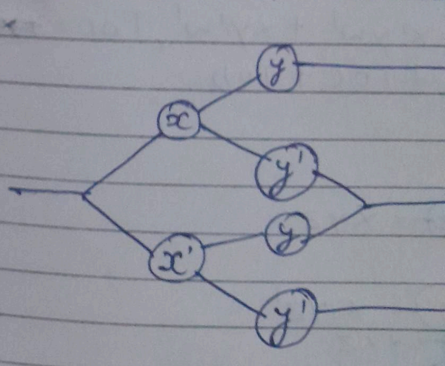
let a = 1st door is opened
b = 2nd door is opened
c = switch of desk board is pressed
x = is the out i.e the bulb is on



EX2) A committee has three member and arrangement is made so that each member has 'Yes' bulb and 'NO' button for voting. A green bulb is on when all are voting and majority is in favour. A red bulb is on when all are voting and majority is in opposition. A yellow bulb is on when all are not voting if any of the member press his vote button and no bulb is on but a bell sound loudly draw the diagram for its control path

Binomial NET :- A n... is always a flow ... called Binomial net ...

⇒ Draw a binomia...



TOA = xy TOC ...

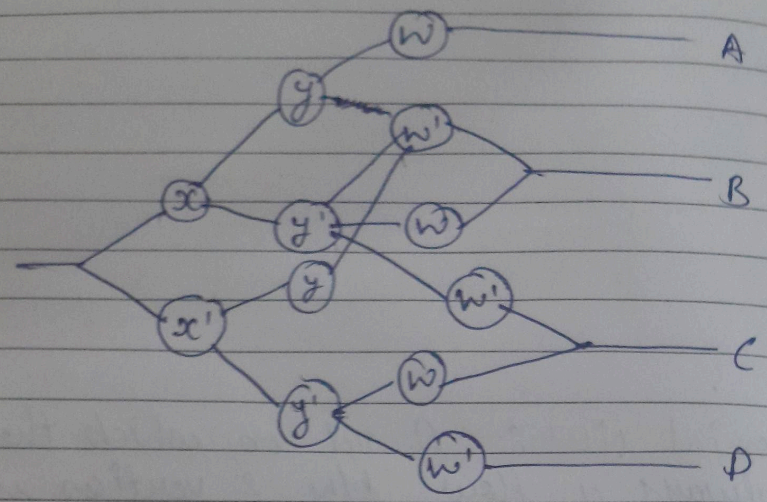
TOB = $xy' + x'y$ T= ...

f = TOA + TOB + TOC

= $xy + x'y + xy' + x$...

= 1

$\Rightarrow$ Draw a binomial net of three variable



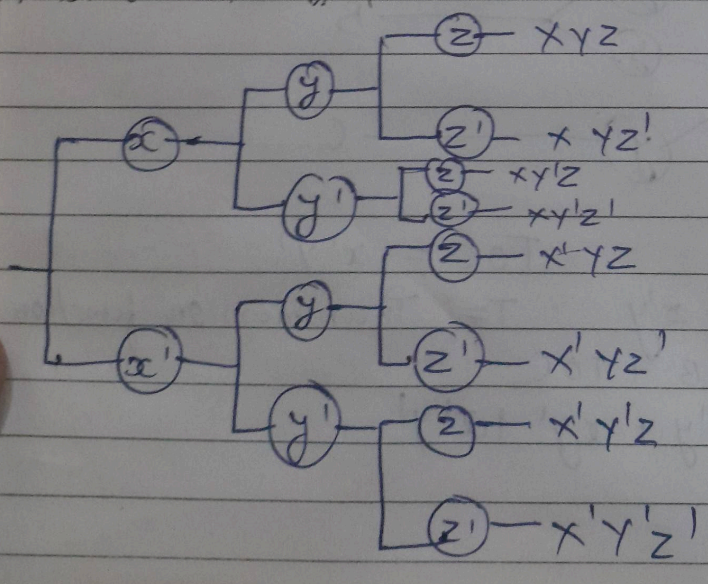
$TOA = xyA$

$TOB = xy'w + xyw' + x'yw'$

$TOC = x'y'w + x'yw' + xy'w'$, $TOD = x'y'w'$

$f = TOA + TOB + TOC + TOD$

Binomial Tree



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$\Rightarrow$ Lower Bound :- let $x = 50$ $y = 20$

then $z_i \sqcap 50 = \{50, 25, 10, 5, 2, 1\}$

$z_j \sqcap 20 = \{20, 10, 5, 4, 2, 1\}$

$z_i \sqcap z_j = \{10, 5, 2, 1\}$

$\Rightarrow$ Greatest lower bound = 10

$\Rightarrow$ upper bound = let $x = 10$, $y = 4$

then $z_i \sqcap 10 = \{10, 20, 50, 100\}$

$z_j \sqcap 4 = \{4, 20, 100\}$

$z_i \cap z_j = \{20, 100\}$

$\Rightarrow$ least upper bound = 20

$\Rightarrow$ ~~lower~~ upper Bound :- let $x = 25$, $y = 10$

then $z_i \sqcap 25 = \{25, 50, 100\}$

$z_j \sqcap 10 = \{10, 50, 20, 100\}$

$z_i \cap z_j = \{50, 100\}$

$\Rightarrow$ ~~Greatest~~ least ~~lower~~ upper bound = ~~100~~ 50

$\Rightarrow$ ~~upper~~ lower bound :- let $x = 25$, $y = 10$

$z_i \sqcap 25 = \{25, 5, 1\}$

$z_j \sqcap 10 = \{10, 5, 2, 1\}$

$z_i \cap z_j = \{5, 1\}$

Greatest lower bound 5

Properties

① A set m...

bound or...

② If the...

are uniq...

③ If a...

for every...

is call...

Q) Draw a...

$x =$ ...

$x \leq$ ...

(i) LOB ...

(ii) the L...

(iii) LOB

20

5, 2, 1}

4, 2, 1}

Y = 4

50, 100}

0 }

Y = 10

}

0}

50

10

Properties

1. A set may have no upper & lower bound or it may have many
2. If the meet and join exist they are unique
3. If a Poset has GLB and LUB for every pair elements then it is called lattice

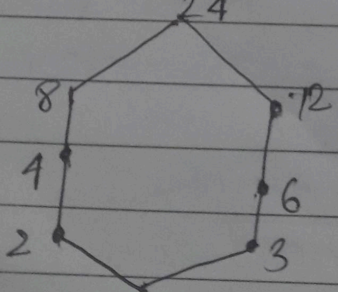
Q) Draw a Hasse diagram for $(X, \leq)$ $X = \{2, 3, 6, 12, 24, 36\}$ and $x \leq y$ if x/y find

(i) LUB and glb of $A = \{2, 3, 4\}$

(ii) the LUB and glb of $B = \{2, 3\}$

(iii) LUB and the Gub of $C = \{6, 12\}$

Q2) for hasse diagram



(i) find lb of 8 & 12
(ii) find ub of 8 & 12
(iii) glb of 8 & 12
(iv) LUB of 8 & 12

Join Semi
In a pos
exist for
then pos
lattice

⇒

c

is not the
not this

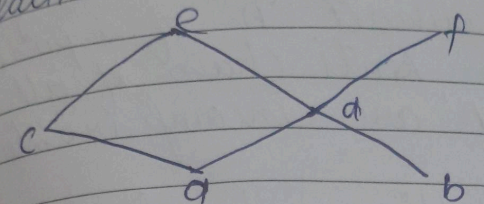
Total ord
partial
S are
(i) compara
(ii) incompa
If
compar
Total
a po

⇒ Well o
a we
poset
orderi

Join Semi lattice

In a poset if lub/Join/Supremum/V(or) exist for every pair of element then poset is called join semi lattice

=>



it is neither join and meet semi lattice as join of e & f is not there and meet of a & b is not there

01/03/2023

Total ordering :- S is a set with partial ordering so the elements of S are

(i) comparable → (3, 9)
(ii) incomparable → (5, 7)

If all the pairs in relation are comparable then it is known as Total ordering provided set S is a poset

=> Well ordered set :- (S, ≤) this is a well ordered set if it is a total poset such that '≤' is a total ordering and every non empty subset

of 8 & 12
b of 8 & 12
8 & 12
8 & 12

of S has atleast one element

Chains & Anti chains :- A subset 'A' of a poset is called a chain if all the element of 'A' are comparable.

A subset 'A' of a poset is called an Anti chain if all the elements of 'A' are incomparable

Distributed lattice :- The lattic 'P' denoted by $(P, \wedge, \vee)$ is distributed lattice if it hold distributed law

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

and

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

$$\forall\, a, b \in A$$

Compliment lattice :- Let $(L, \wedge, \vee)$ is a lattice and $0, I \in L$ such that $0 \leq a \leq 1 \;\forall a \in L$ then

$$a \vee 1 = a \quad , \quad a \wedge 1 = a$$

$$a \wedge 0 = 0 \quad , \quad a \vee 0 = a$$

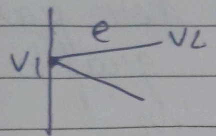
Now if for $a \in L$ there is $a' \in L$ such that $a \wedge a' = 0$, $a \vee a' = 1$ then a' is called complement of a and lattice is called compliment lattice

Graph
↳ It is
G(V, E
↳ vertex/nod
↳ Edge/ Line
↳ degree o
↳ this is
↳
every graph
↳ degree
↳ degree o
⇒ Graph :-
a
V = {
v's el
point)
called
→ Self
a
when
same

Graph

↳ It is collection of vertex & edges $G(V, E)$

↳ vertex/node/point = $\{v_1, v_2 \dots v_n\}$
↳ Edge/Line/Branch = $\{e_1, e_2 \dots e_n\}$



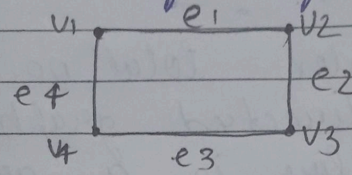
↳ degree of vertex = total no. of edges
↳ this is undirected graph
↳ Every tree is a graph but every graph ~~tree~~ may or may not be a tree
↳ degree of root = 0 (in tree)
↳ degree of vertex = 1 (in tree)

⇒ Graph:- A graph $G(V, E)$ consist of a set of objects
$V = \{v_1 \dots v_n\}$, $E = \{e_1, \dots e_n\}$
V's element are called vertices (or point/Node) and E's elements are called edges (lines/branches)

+ Self loop:- An edge is said to be a self loop (or simply a loop) when its both the vertices are same

→ **Parallel edges** :- If there are two or more than two edges having the same pair of end vertices then such edges are called parallel edges or multiple edges

→ **Simple graph** :- A graph that has neither self loop nor parallel edges is called simple graph



→ **Multi / pseudo graph** :- A graph that has self loop or parallel edges is called multi/pseudo graph

→ **Incident edge** :- let V_i be an end vertices of some edge e_j then we can say that edge e_j is incident on the vertex V_i

for example :- e1, e3 and e4 are incident on V2

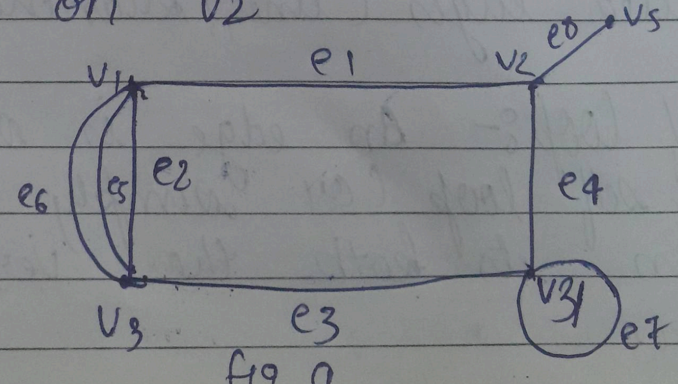


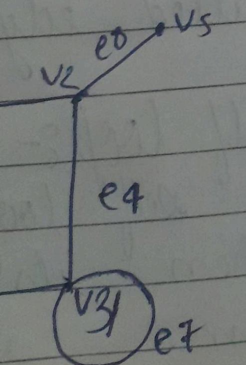
fig 9

...re are two or
...es having the
... vertices then
... called parallel
... edges

... that has
... parallel edges
...aph

A graph that
parallel edges
graph

...i be an end
...ge ej then
edge ej is
...x v_i
... and e4 are



* Adjacent edges :- Two non parallel edges are call adjecent if they are incident on common vertex

* Adjacent vertices :- Two vertices are called adjacent if there is an edge join them it means v2 and v4 are adjacent vertices (from fig a)

* Degree of vertices :-
The degree of vertices v_i is denoted by degree v_i (see fig a)

degree $(v_1) = 4$
degree $(v_4) = 4$ find same for all

Adjacent edges :- Two non parallel edges are call adjecent if they are incident on common vertix

Adjacent vertices :- Two vertices are called adjacent if there is an edge join them it means V2 and V1 are adjacent vertices (from fig a)

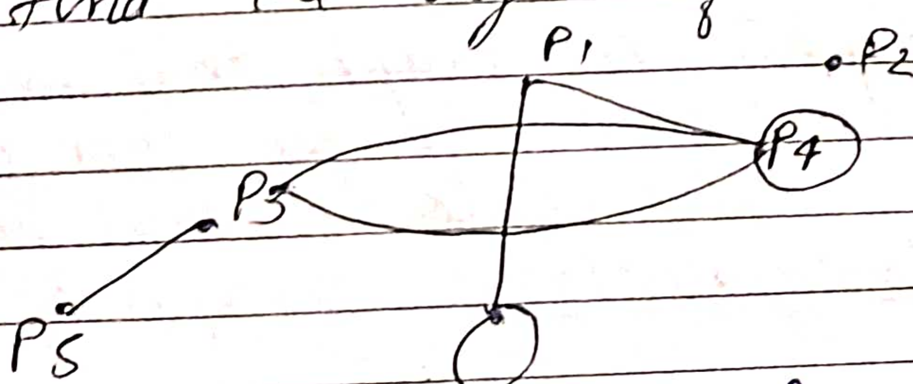
Degree of vertices :-
The degree of vertices V_i is denoted by degree V_i (see fig a)
degree $(V_1) = 4$
degree $(V_4) = 4$ find Same for all

02/03/2023

Regular graph :- A graph G in which all vertices are of equal degree is called regular graph

ex:- 

find the degree of each vertex



deg $(P_1) = 2$ deg $(P_2) = 0$, deg$(P_3) = 3$
deg $(P_4) = 5$ deg $(P_5) = 1$, deg $(P_6) = 3$

→ Null graph :- The null graph is a graph containing no edges, the null graph with n vertices is denoted by N_n

ex :- $\cdot N_1$ $\cdot N_3$

$\cdot N_2$

degree is 0 in null graph for each vertex

→ Isolated vertex :- A vertex with 0 degree is called isolated vertex.

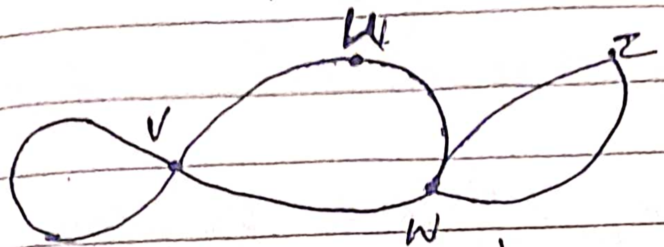
→ Pendant or Trivial graph :- A vertex of degree one is called pendant graph.

→ Finite & infinite graph :- A graph $G(V,E)$ is finite if both V & E are finite. The graph $G(V,E)$ is infinite if both V & E are infinite

→ Connected graph :- A graph that is in one piece is said to be connected whereas one split into several pieces is called disconnected graph

→ Sub graph :- Subgraph of a graph G is a graph whose all the vertices belongs to $V(G)$ and all whose all the edges belongs to $E(G)$

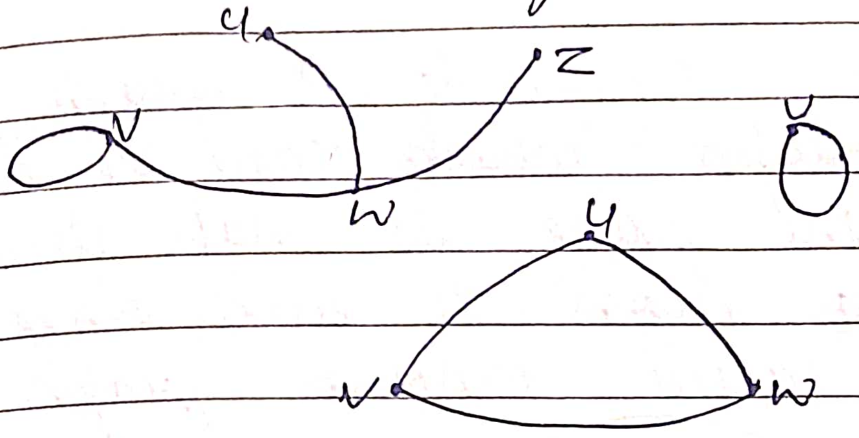
for example if G is the connected graph



where $V(G) = \{v, u, w, z\}$

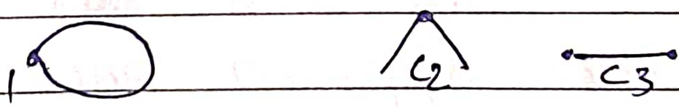
$E(G) = \{vv, uw, vw, wz, wz\}$

then the following graphs are subgraphs



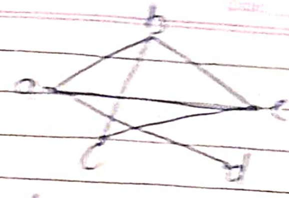
\# Cycle graph:- A cycle graph is a graph of single cycle. The cycle graph with n vertices is denoted by C_n

ex:- C_1



\# Handshaking Theorem

~~Consider a gex~~ The sum of all the degrees of a graph is equal to twice the number of edges.



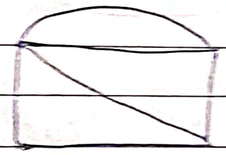
Here vertex $V = \{a, b, c, d, e\}$

edges $= \{ab, ad, ae, bc, be, ce, de\}$
$= 7$

$deg(a) = 3$, $deg(b) = 3$, $deg(c) = 2$
$deg(d) = 2$, $deg(e) = 4$

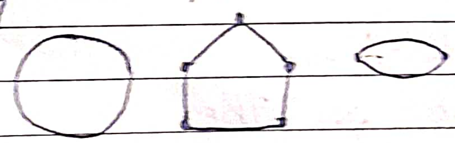
→ Planar graph :- A graph or multigraph which can be drawn in the plane so that its edges do not cross to each other, then it is called PLANAR Graph.

ex :-



→ Circuit :- The close ~~Box~~ walk in which no vertex appears once is called circuit

ex:-



→ Distance :- The distance $d(v_i, v_j)$ b/w any given pair of vertex v_i & v_j

is defined
path that
the shorte

→ Eccentricity
vertex
distance
furest
matically
where

→ Centre:-
minimum
as the

a

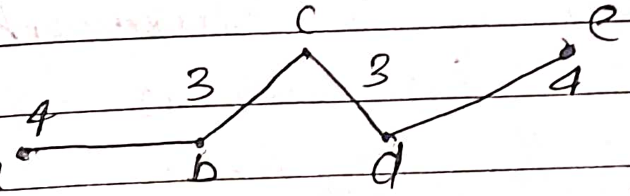
$E(a) =$
$E(d) =$
there

→ In
G the
of G
D
is the
vertex
then

is defined to the length of shortest path that is the no. of edges in the shortest path

- Eccentricity :- The eccentricity E of a vertex v is defined as its max distance from any other vertex farest from E this can be mathematically as $E(v) = \max d(v_i, v_j)$ where $i = 1, 2, \ldots n$.

- Centre :- A vertex in a graph G with minimum eccentricity is refered to as the centre of G



$E(a) = 4$, $E(b) = 3$, $E(c) = 2$
$E(d) = 3$, $E(e) = 4$.
these are the eccentricity values

03/03/2023

- In & Out degree :- In directed graph G the Out degree of vertex v of G is denoted by $deg_G^+(v)$ or $deg_G(v)$, it is the no. of edges ~~in the~~ ~~directed~~ ~~graph~~ ~~G~~ of ~~the~~ ~~vertex~~ beginning at v and then in degree of vertex is denoted

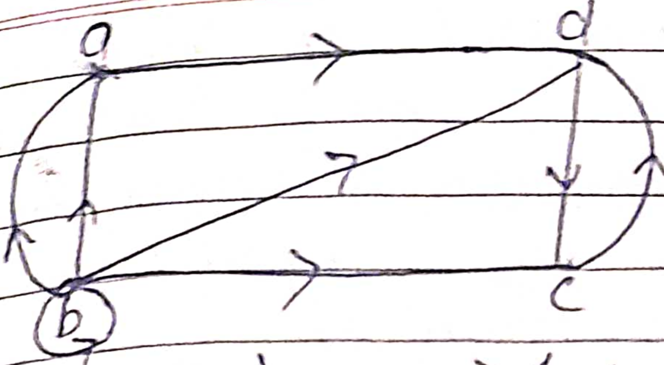
Date: / / Page no:

if the no. of edges ending at v
the sum of in and out degree of a vertex is called the total degree of the vertex
The vertex with '0' in degree is called source and a vertex with '0' out degree is called sink
if G(V, E) be a directed graph with e edges

$$\sum_{v \in V} \deg_G^+(v) = \sum_{v \in V} \deg_G(v)$$

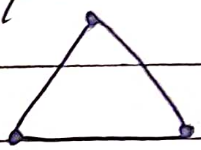
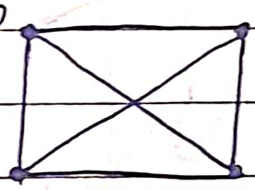
it means the sum of out degrees of the vertices of a digraph equals the sum of in degrees of the vertices which equals the number of edges in G.

Proof:- Any directed edge (u, v) contributes 1 to the degree of v and 1 to the out degree of v. The loop at v contributes 1 to the in degree and 1 to the out degree of v
hence proved



⇒ Indegree (a) = 2 ⇒ Indegree (b) = 1
outdegree (a) = 1 , outdegree (b) = 5
⇒ Indegree (C) = 2 ⇒ Indegree (d) = 3
outdegree (C) = 1 , outdegree (d) = 1

→ Complete Graph :- A simple graph G is said to be complete if every vertex in G is connected with every other vertex. If G contains exactly 1 edge b/w each pair of distinct vertices a complete graph usually denoted by "K_n" and it should be noted that K_n has exactly $n(n-1)/2$ edges.

ex:-  

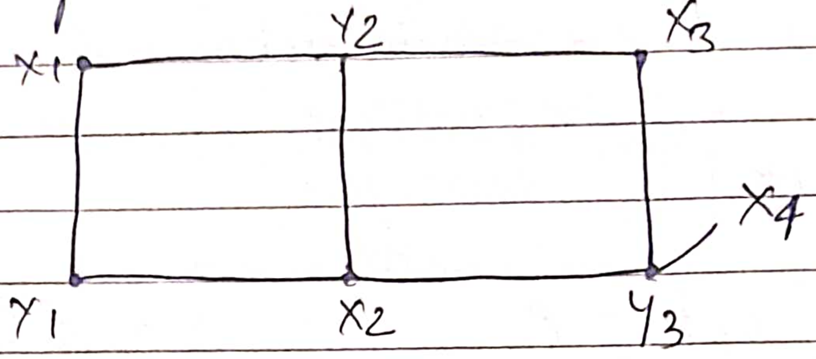
→ Wheel Graph :- A wheel graph (W_n) ($n>3$) is obtained from C_n by adding a vertex v inside C

→ Bipartite Graph :- A graph G(V, E) is Bipartite if the vertex set V

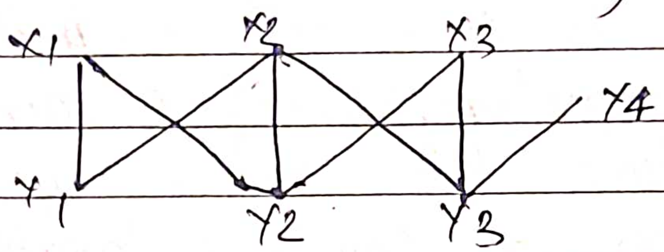
Date: / / Page no.

can be partitioned on the two subset (V_1 & V_2) such that every edge in e connects a vertex in V_1 and a vertex V_2, V_1 and V_2 is called a bipartition in G. A Bipartite graph doesn't have a loop.

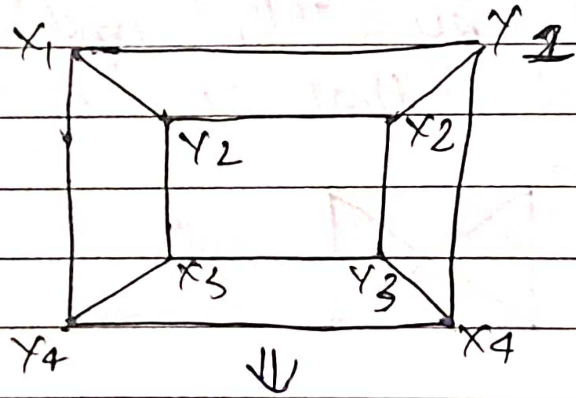
(i)



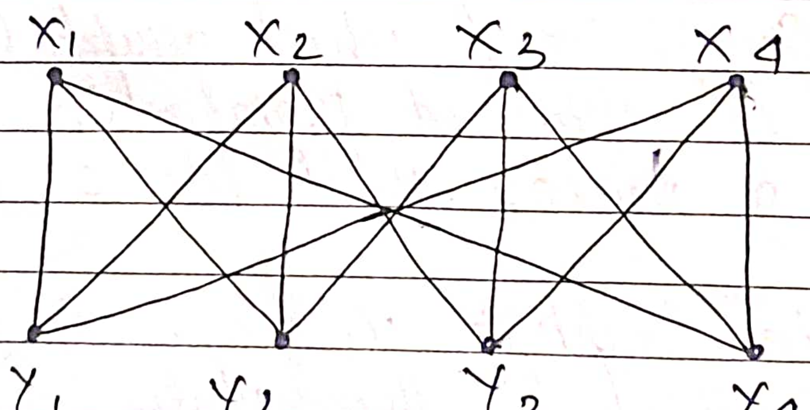
⇓ Converting it into Bipertite



⇒



⇓



Date: / / Page no:

→ Complete Bipartite The complete Bipartite graph on m & ~~n~~ n vertices denoted by K_{mn}, is a graph whose vertex set ~~its~~ is partition into sets V_1 with m vertices & V_2 with n vertices in which there is an edge b/w each pair of vertices v_1 & v_2 where v_1 is in V_1 and v_2 is in V_2

⇒ Any graph in the form of $K_{1,n}$ is called star graph.

⇒ A complete Bipartite graph K_{mn} is not regular graph